INSTRUCTIONS
Written Ph.D. Preliminary Examination
and Master's Qualifying Examination
in Demographic Methods

Department of Demography
University of California, Berkeley
Monday, 6 May 2019, 10:00 A.M. to 1:00 P.M.

This is a closed-book examination. You may bring a dictionary (especially for non-native speakers of English) and a calculator, but do not bring a smart phone, papers, books, computers, or mobile devices. A sheet of Useful Formulas is supplied with the exam.

All four questions count equally. You will be assigned a code in place of your name. Please write your answers on 8.5" by 11" white paper, starting each question on a new page, with your code and page number on the upper right of each sheet, writing on only one side of each sheet. Do not staple. (Your exams will be fed through a scanner for backup.) Answers with decimals should be given with six figures beyond the decimal point. Please label your answers and be clear.

A Collection of Useful Formulas

Growth Rate: $R = (1/T)\log(K(T)/K(0))$

Exponential Growth: $K(t) = K(0)e^{Rt}$

Interval Conversions: $(1 - {}_1q_x)^n = 1 - {}_nq_x$

Exponential Slope: $\frac{d}{dx}\alpha e^{\beta x} = \alpha\beta e^{\beta x}$

Exponential Area: $\int_a^b \alpha e^{\beta x}dx = (\alpha/\beta)(e^{\beta b} - e^{\beta a})$

Age Specific Death Rate: ${}_nM_x = {}_nD_x \,/\, {}_nK_x$

Period Lifetable: ${}_nq_x = \frac{(n)({}_nM_x)}{1 + (n - {}_na_x)({}_nM_x)}$

Age Factors: ${}_1a_0 = 0.07 + 1.7({}_1M_0)\,;\quad {}_4a_1 = 1.5\,;\quad {}_\infty a_x = 1/({}_\infty M_x)$

Survivorship: $\ell_{x+n} = \ell_x(1 - {}_nq_x) = \ell_x - {}_nd_x$

Person-Years Lived: ${}_nL_x = (n)(\ell_{x+n}) + ({}_na_x)({}_nd_x)$

Lifetable death rate: ${}_nm_x = {}_nd_x/{}_nL_x$

Remaining Person-Years: $T_x = {}_nL_x + {}_nL_{x+n} + {}_nL_{x+2n} + \cdots$

Expectation of Life: $e_x = T_x/\ell_x$

In the Presence of Other Causes: ${}_nq_x^A = ({}_nq_x)s_A$

In the Absence of Other Causes: ${}_nq_x^{A*} = 1 - (1 - {}_nq_x)^{s_A}$

Annuity Price: $\frac{B}{\ell_x}\left(\frac{{}_nL_x}{(1+i)^{n/2}} + \frac{{}_nL_{x+n}}{(1+i)^{n+n/2}} \ldots + \frac{{}_\infty L_{xmax}}{(1+i)^{xmax-x+e_{xmax}}}\right)$

Brass's Logit System: $\ell_x = \frac{1}{1 + \exp(-2\alpha - 2\beta Y_x)}$

Brass Estimation: $\alpha + \beta Y_x = (1/2)\log(\ell_x/(1-\ell_x))$

Hazard Rate: $h_x = -(1/n)\log(\ell_{x+n}/\ell_x)$

Survival from hazards: $\ell_{x+n} = l(x)e^{-h_x\ n}$

Cumulative Hazard: $H_x = -\log(\ell_x/\ell_0)\quad ; \quad \ell_x = \ell_0 e^{-H_x}$

Deaths from Hazards: ${}_nd_x = \int_x^{x+n} h_a \ell_a da = \int_x^{x+n} h_a e^{-H_a} da$

Gompertz Model: $h_x = \alpha\, e^{\beta x}\quad ; \quad \ell_x = \exp\big((-\alpha/\beta)\,(e^{\beta x} - 1)\big)$

Gompertz Modal Age: $x_{mode} = (1/\beta)\log(\beta/\alpha)$

Proportional Hazards: $h_t(i) = h_t(0)\exp\left(\beta_1 X_1(i) + \beta_2 X_2(i) \ldots\right)$

Leslie Matrix Top Row: $\frac{{}_nL_0}{2\ell_0}\left({}_nF_x + {}_nF_{x+n}\frac{{}_nL_{x+n}}{{}_nL_x}\right) f_{fab}$

Leslie Matrix Subdiagonal: $\frac{{}_nL_{x+n}}{{}_nL_x}$

Lotka's Continuous-Age Equation: $1 = \int f_x \, \ell_x \, (f_{fab}/\ell_0) e^{-rx} \, dx$

Lotka's Equation: $1 = \sum (1/2) \left({}_nF_{x n}L_x + {}_nF_{x+n n}L_{x+n} \right) (f_{fab}/\ell_0) e^{-r(x+n)}$

Stable Age Pyramid : ${}_nK_x^{stable} = B({}_nL_x/\ell_0) \, e^{-rx}$

Stable Proportions ${}_nK_x/{}_\infty K_0 = b \, ({}_nL_x/\ell_0) \, e^{-rx}$

Lotka's Parameter: $r \approx \log(NRR)/\mu$

NRR and GRR: $NRR \approx GRR \ \ \ell_\mu$

Momentum: $\frac{K(+\infty)}{K(-\epsilon)} = \frac{b(-\epsilon)e_0}{\sqrt{NRR}}$

Coale-Trussell Model: ${}_nF_x^{marital} = Mn(x)exp(-m\nu(x))$.

Coale-Trussell Estimation: $\log({}_nF_x^{marital}/n(x)) = \log(M) - m\nu(x)$.

Parity Progression Ratios: $PPR(j) = \sum_{j+1}^{\infty} w(i) \, / \, \sum_{j}^{\infty} w(i)$

Renewal Equation: $B(t) = \int B(t-x) \, \ell_x(t-x) \, f(x,t) \, f_{fab} \, dx$

SMAFM $= \sum (n) \, (1 - PEM_x/PEM_{ult})$

Bourgeois-Pichat's Formula ${}_dq_0 = a + b(\log(1+d))^3$

Princeton Indices: $I_f \approx I_g I_m$

$$I_f = \frac{B^{overall}}{\sum ({}_5K_x)({}_5F_x^{Hutt})}$$

$$I_g = \frac{B^{marital}}{\sum ({}_5K_x^{married})({}_5F_x^{Hutt})}$$

$$I_m = \frac{\sum ({}_5K_x^{married})({}_5F_x^{Hutt})}{\sum ({}_5K_x)({}_5F_x^{Hutt})}$$

Linear Regression slope: $= \dfrac{mean(X*Y) - mean(X)*mean(Y)}{mean(X*X) - mean(X)*mean(X)}$

Linear Regression intercept: $= mean(Y) - (slope)*mean(X)$

Duncan Dissimilarity Index $D = \max\left(\dfrac{\sum_1^J U_{(j)}}{\sum_1^n U_{(j)}} - \dfrac{\sum_1^J R_{(j)}}{\sum_1^n R_{(j)}}\right)$